PhD thesis on —
Formal methods for the control of switching systems

Supervisors: Antoine Girard (antoine.girard@l2s.centralesupelec.fr)
Thiago Alves Lima (thiago.alveslima@l2s.centralesupelec.fr)

Location: Laboratoire des Signaux et Systèmes - L2S
CentraleSupélec – CNRS – Univ. Paris-Saclay
91190, Gif-sur-Yvette, France

General information: Fully funded PhD, 36 months, starting in **autumn 2024**.

Application details: Interested candidates will email the supervising team enclosing their CV, cover letter, and other relevant documents (reference letter(s), research statement, bachelor's/ master's official transcripts, etc). Selected applicants will be invited to an interview.

Deadline for application: 10 May 2024.

Context

Switching systems represent a type of hybrid/cyber-physical dynamical system where the system behavior depends on a finite set of N subsystems and on a switching function σ selecting which subsystem is active at any given moment [1, 2]. More formally, considering matrices $A_1, \ldots, A_N \in \mathbb{R}^{n\times n}$, $B_1, \ldots, B_N \in \mathbb{R}^{n\times m}$, and $C_1, \ldots, C_N \in \mathbb{R}^{p\times n}$, in this thesis the candidate will study the *discrete-time switched system* defined by

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \\ y(k) &= C_{\sigma(k)}x(k) \end{aligned} \tag{1}$$

where $\sigma : \mathbb{N} \to \{1, \ldots, N\}$ is a switching signal selecting among the N modes of the system (1), $u : \mathbb{N} \to \mathbb{R}^m$ is a control input, and $y : \mathbb{N} \to \mathbb{R}^p$ is the system's output.

Problems related to the stability and stabilizability of discrete-time switching systems under different scenarios on the switching signal have been an active field of research due to their important applications in modern engineering [3, 4]. One example is network control systems, where transmission delays can be modeled by switching systems. When studying switched systems, one common assumption is to consider the case of *arbitrary switching signals* σ. The stability of (1) under no external inputs (i.e., $u \equiv 0$) is, in this case, related to the notion of joint-spectral radius (JSR) of the matrices A_i [5]. Recently, the stabilization of discrete-time switched systems under *arbitrary switching signals* using *piecewise linear* control laws and graph-based Lyapunov functions was studied in [6]. In [7], optimal stabilizing decay rates also under this class of switching signals were explored.

Considering arbitrary switching rules can, however, be conservative since requiring that *all the solutions* are driven to the equilibrium, for *all the possible switching sequences*, is a demanding (and often unfeasible) task. Moreover, in several real-life situations, the considered systems present some sort of constraints in the admissible switching events. For this reason, it is rather common, both for stability and stabilizability purposes, to consider subclasses of switching signals. Many questions regarding these subclasses of switching signals remain open, mainly due to their inherently more complicated nature. In this context, the framework of language and automata theory naturally arises as a valuable tool, providing graph-theoretic tools to encode constraints on the switching signals.

Some notions regarding the *stability* of switched systems with switching signals modeled by ω-regular languages have been explored in recent works [8, 9]. For *stabilization* purposes, that is, designing the input signals u, a very specific class of ω-regular language that Büchi automata can model was recently explored in [10], allowing the stabilization of systems with non-controllable pairs (A_i, B_i). Nonetheless, the case of stabilization under general or other subclasses of ω-regular languages has not been considered. The advantage of using ω-regular languages is that they can represent intricate behavior of switching signals, such as the fact that some subsequence of modes may be repeated an infinity of times without a specific frequency. These intricate behaviors can be modeled with the help of formal methods from computer science and thus offer a powerful framework for the control of (1).

Objective and work description

In this PhD, the selected candidate will explore the control of discrete-time switched linear control systems under different assumptions on the switching signals. The work will start with a detailed review of recent contributions related to the case of arbitrarily varying switching signals, especially by understanding the techniques developed in [10, 6], which employ graph-theoretical approach and Büchi automata for computing stabilizing *feedback* control laws. The natural sequence of the work will be to explore the stabilization of switched systems under different classes of $\omega - regular$ constrained switching signals. Therefore, more general classes of languages than that of [10] will be considered. In particular, formal methods from computer science will be essential. Different types of automata, such as Rabin automata, will be needed. The advantage of Rabin automata over Büchi automata is that a *deterministic* Rabin automata can model any ω-regular language since any LTL formula can also be translated into a language-equivalent deterministic Rabin automaton [11, p. 79]-[12]. The same only holds for *non-deterministic* Büchi automata. Besides the design of state-feedback controllers, constructing up on the results of [13], observer-based output feedback controllers may also be explored. Finally, we also aim to study optimal stabilization rates under such classes of constrained switching signals, extending the results from [7] to the studied classes of switching signals.

Candidate profile

The ideal candidate for this PhD project should possess a Master's degree (or equivalent) in mathematics or engineering, with a preference for a specialization in control systems. They should have a robust mathematical foundation and demonstrate excellent English writing and speaking skills. Given that L2S operates on an international stage, proficiency in French is not a prerequisite. Experience with software tools like MATLAB/Simulink for modeling, simulation, and testing is advantageous, though candidates will have the opportunity to acquire and refine these skills during the initial months of the PhD program.

Advantages

The selected candidate will thrive in a vibrant, international environment that fosters research and cultural exchanges. L2S hosts numerous seminars in control systems and regularly welcomes visitors from across the globe, enriching our academic community. Exceptional progress in their PhD research may allow candidates to spend some months at overseas laboratories, under the guidance of our collaborating scholars. Furthermore, participation in international conferences is integral to the PhD journey, offering unparalleled exposure and networking opportunities.

References

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