Switched systems are systems involving both continuous and discrete dynamics. They consist of a finite number of subsystems and a discrete rule that dictates switching between these subsystems. They have been widely studied during these two last decades because they can describe a wide range of physical and engineering systems [1].

Further, in mathematical control theory, most of the existing methods can only be applied to systems evolving either on the continuous-time domain [2] or on the discrete uniform time domain [3]. The extension of these results to switched systems evolving on a non-uniform time domain is still an open problem. In engineering or in several areas of industry, there are many switched dynamical systems that evolve on a non-uniform time domain that can be discrete with non-uniform sampling or a combination of discrete and continuous time domains (see Fig. 1.a). There are several applications involving such systems (see Fig. 1.b). Impulsive systems (which are a relevant class of switched systems, in which the state jumps occur only at some time instances) with non-instantaneous state jumps are other examples. Indeed, the temporal nature of previously introduced systems cannot be represented by the real line (i.e. R) or discrete line (i.e. Z). Cooperative control over network has also attracted a great deal of attention in the last few years due to its broad range of applications in many areas (flocking, consensus [4], see Fig. 2.). Existing controllers and observers can be categorized into two separated directions depending on whether the agents are described via continuous-time or discrete-time models. Usually, the existing works on formation control or coordination control assume that relative local information among agents is transmitted continuously or at some moments with an identical step size. However, this assumption is unrealistic due to, for instance, unreliability of communication channels, external disturbances and limitations of sensing ability. Indeed, local information is exchanged over some disconnected time intervals, due to bandwidth limitations, communication obstacles or sensor failures. Therefore, it is of practical interest to consider the case of intermittent information transmission among neighbor agents. In this case, the time domain is neither continuous nor uniformly discrete, due to possible intermittent information transmissions for instance.

Engineers have to face the new challenges driven by non-uniform time domain and in particular, to deal with the estimation on an arbitrary time domain. A wide variety of information cannot be directly obtained through a measurement. Thus, it is needed to extract the information conveyed by the signals in order to estimate (when possible) the missing information. These observers can be viewed as a cheap but efficient tool allowing variables of interests to be recovered from available measurements.
The objective of this thesis is to analyze the observability and to design observers for switched systems evolving on a non-uniform time domain using the time scale theory. It should be highlighted that many results are available in the literature using either the theory of dynamical systems in continuous time or discrete time dynamical systems. This observation raises two interesting questions:

- Is it possible to unify the theories for systems defined on continuous time domains with that for switched systems on discrete time domains?
- Is it possible to extend the existing theory to switched systems on generalized hybrid (continuous/discrete) domains?

Stephan Hilger showed, in 1988, that a broader theory, entitled “time scale theory” may give positive answers of the two previous questions. This theory was introduced in order to unify the theory of difference equations and differential equations [5]. First, what is a time scale? It is a non-empty closed subset of the real number \( \mathbb{R} \), provided with an induced topology of \( \mathbb{R} \). The most classical time scales are those that represent the real time domain \( T = \mathbb{R} \), on which the continuous dynamical systems are studied, the time scale that represent the discrete time domain \( T = h\mathbb{Z} \) (\( h \) is a positive constant) on which one studies the discrete dynamical systems etc. The interest of the time scale theory lies in the arbitrary choice of the time scale. Indeed, we can also choose a mixture of discrete points and closed intervals for the time scale. Further, once a result has been established for dynamic equations on an arbitrary time scale, this result holds for standard continuous differential equations (i.e. \( \mathbb{R} \)) and standard difference equations (i.e. \( h\mathbb{Z} \) with \( h \) is a positive constant). Though there exist some results on dynamic equations evolving on non-uniform time domains, control and observation theory on time scales is not much developed. The thesis will introduce innovative principles based on time scale theory to unify the observation issues for switched systems defined on continuous time domains with that for systems on discrete time domains and to extend the existing results to dynamical systems on generalized hybrid domains.

In our previous works, the following problems were discussed, and several results were proposed and published.

- Stability analysis of switched linear system on non-uniform time domain
  - Using upper bound of the solution [6] assuming pairwise commuting matrices
  - Using common Lyapunov functions [7] without assuming pairwise commuting matrices

- Control of multi-agent systems on non-uniform time domain
  - Using the time scale theory [8-10]
Using common Lyapunov functions [7]
The objective of this thesis is to derive new results on observability and observer design using the time scale theory.

Research subject and work plan:

The purpose of this thesis is the design of new theoretical tools for the observability analysis and observer design for switched systems on non-uniform time domain using the time scale theory. An application to the consensus problem and coordination under intermittent information transmissions is also envisaged.

The main further works will concern:
- To analyze the observability of switched systems on non-uniform time domain using the time scale theory. This property has been widely studied in the continuous and discrete time cases, leading to closed results for both cases. Very recently, based on the time scale theory, a new method has been proposed to relate the results in one case (continuous or discrete) with the results in the other. Indeed, for a class of linear time-invariant systems on time scales, it was shown that the standard Kalman criterion of observability holds. For nonlinear systems, the main problem is to find necessary and sufficient conditions for an abstract input/output map to have a realization as a nonlinear system on the time scale. Here, the research will be started from the simplest linear case, and then extend to treat the nonlinear case.
- To design observers for switched systems on non-uniform time domain using the time scale theory. In many practical systems, it is needed to know the system state for the control or the diagnostic for instance. However, sometimes, in practice, the system state is not fully measurable. Indeed, it is important to note that the use of physical extra sensors in order to measure the state increases the cost and the complexity of the system. Therefore, its estimation by means of observers becomes an attractive and economical solution. Different observer-based methods have been developed for nonlinear system in the continuous and discrete cases. The question is how we can unify and extend the existing results to systems evolving on generalized hybrid domains. It should be highlighted that the possible non-uniformity of the time domain creates obstacles in the design objective. The time scale can be a priori known in some future window or only a stochastic knowledge of the distance between sampling points can be assumed. Here, the research will start on the observer design for linear system assuming that the time scale is known a priori, and then extend to more general cases.
- To apply these techniques to networked control systems. The problem of consensus with intermittent information transmissions can be converted to the asymptotic stabilization problem for a particular switched system on a non-uniform time domain. Indeed, the interaction among agents happens during some continuous time intervals with some discrete time instants. Therefore, it is of high interest to mix the continuous-time and discrete-time cases under a unified framework. The theory of system dynamic on an arbitrary time scale seems to be very promising to solve this problem. Another possible extension is to study self-triggering strategy to relax the usual fixed sample rate assumption allows scheduling control. It enables to reduce the transmitted information.

Requested theoretical knowledge

- Mathematics: graph theory, multi-agent systems
- Automatic control: Dynamics and control of nonlinear systems

Requested programming knowledge:

- Candidates should be experienced in the use of MATLAB, C or C++ Language programming

Working context and environment

This subject is jointly proposed by the department of Automatic Control of LAMIH research laboratory (France) and by the University of L’Aquila (Italy)
References: