Title: Efficient and Reliable Control of coupled Stochastic and Partial differential Equations (ERC–SPE)

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Description of the Thesis:

I. Context

Stochastic differential equations (SDEs), which are coupled with (possibly non-linear) partial differential equations (PDEs), are a class of systems that naturally arise when modeling processes whose dominant probabilistic dynamics are affected by destabilizing second-order convection-diffusion-reaction effects. We refer to such control systems as SDE+PDE systems.

A relevant example of a SDE+PDE system is provided by District Heating and Cooling Systems (DHCSs) [1, 2]. DHCSs deliver thermal energy to a network of buildings from an outside source. They offer numerous advantages over individual building apparatus, including greater safety and reliability, reduced emissions, and reliance on alternative fuels such as biomass or waste. However, due to low operating temperatures and limited flow capacity, customer demand can be met only if: 1) non-linear SDE-based dynamical models are leveraged to capture as many uncertain weather fluctuations as possible, and 2) the aforementioned stochastic models are coupled with sophisticated non-linear PDEs to accurately capture energy losses which often occur along pipelines and hinder performance. In turn, the overall setting require to efficiently and reliably control a complex SDE+PDE system.

Unfortunately, still too few and specific works on SDE+PDE systems appear in the literature, hence calling for the design of novel tools for the efficient modelization and control of such systems.

Thesis goal: Design methods for efficient and theoretically guaranteed control of a broad class of SDE+PDE systems. The proposed approaches will have to be constructive to obtain a semi-explicit design of the corresponding control laws, enabling performance-efficient numerical paradigms.

II. Scientific approach

The main idea of the proposed approach leverages and combines promising and complementary techniques, which have been so far developed independently: statistical linearization [3], and recent methods for the stabilization of coupled linear PDE–ODE systems [4, 5]. On the one hand, statistical linearization allows one to reduce a stochastic control problem to the control of a deterministic nonlinear system. It is obtained by appropriately linearizing the original probabilistic model around the mean and under additional state constraints which are proven to preserve well-posedness. Promising results through the use of this technique have been recently obtained for the optimal control of non-coupled, though sophisticated SDEs [6]. On the other hand, recent backstepping-based methods, finite-dimensional approximation, and Lyapunov-analysis-based techniques have shown promising results for the control of classical ODEs coupled with hyperbolic and/or parabolic PDEs [7, 8, 9, 10]. Although only PDEs-PDEs or PDEs-ODEs interconnections have been considered in the literature so far, we believe these approaches can be extended to interconnections of PDEs with SDEs. Importantly, these methods rely on the inherent regularizing properties of the system and require more investigation. On the other hand, approaches based on representation formulas for 1D hyperbolic equations [11] showed promising results towards the study of control properties of these systems and will have certainly extensions in the case of inerconnected networks of 1D hyperbolic equations.

The scientific objective of the thesis is to first extend and then combine these previous approaches to the case of SDEs coupled with PDEs. The principal steps of the proposed work are listed as follows:

- Investigate under which conditions the well-posedness of statistical linearization can be strengthened, enabling general efficient and reliable numerical methods for the control of SDEs.
- Investigate under which conditions the most recent methods for PDE–ODE can be extended to a broader class of systems, and in particular develop techniques which are as much independent as possible from any inherent regularizing property of the system (e.g., optimization methods).

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- Leverage statistical linearization to transform the SDE defining the SDE+PDE system of interest into a (constrained) well-posed ODE, and leverage the aforementioned improved methods for PDE–ODE systems to design efficient control strategies for the original SDE+PDE system.
- Although the objective of the thesis consists of developing control methods which work in very general settings, we plan to showcase the efficiency of the proposed approaches through numerical simulations on specific examples (for instance, on DHCSs). Implementation on a real robotic system might be considered, which might result in an external collaboration with Stanford University.

III. Required skills

This thesis topic mainly requires good skills in automation and mathematics (Grandes Ecoles or Master in mathematics/control). Very good results in the engineering curriculum as well as expertise in the topics related to automatic and stochastic and partial differential equations will constitute strengths to the proposed subject. The proposed subject shall lead to the acquisition of strong theoretical skills in the field of control of systems described by both stochastic and partial differential equations. In particular, the candidate shall become familiar with the modeling of dynamical systems, with control design, and with both Itô-type and convection–diffusion-reaction equations. The candidate shall also become familiar with Julia, Matlab, or Python (numerical methods, simulations).

IV. References

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