Title: Practical control of simple interconnections of hyperbolic systems

Laboratory: Laboratoire des signaux et systèmes, CNRS, CentraleSupélec, Université Paris Saclay, 91190, Gif-sur-Yvette, France

Internship Advisors: Jean AURIOL, Lucas BRIVADIS[1].

Description of the internship

I. Context and scientific objectives

Distributed parameter systems provide a natural representation of industrial processes involving the evolution of quantities in time and space. In particular, hyperbolic partial differential equations play a crucial role in the mathematical description of transport phenomena with finite propagation speeds, e.g., transport of matter, sound waves, and information. **Networks of hyperbolic Partial Differential Equations (PDEs) systems**, possibly coupled with Ordinary Differential Equations (ODEs), constitute an essential paradigm to describe a wide variety of large complex systems, including wave propagation, traffic network systems, electric transmission lines, hydraulic channels, drilling devices, communication networks, smart structures, multiscale and multiphysics systems [1, 2, 3]. Controlling and monitoring networks of hyperbolic systems are **difficult control engineering problems** due to the **distributed** nature of the different subsystems composing the network (time and space dependency), the possibly involved **graph structure** of the network, and the physical/economic **infeasibility of placing sensors and actuators everywhere** along the spatial domain. The stringent operating, environmental, and economical requirements and the high mathematical complexity of these systems explain why traditional control methods exhibit a limited range of applicability and have not been successful at high technology readiness levels (TRLs) [4, 5]. Thus, the theory of control of distributed parameter systems needs substantial advancements to achieve control and estimation objectives for such network structures.

II. Scientific approach

In this internship, we consider networks composed of interconnected *elementary* hyperbolic subsystems. These different hyperbolic subsystems correspond to one-dimensional linear balance law systems [1, Chap. 5]. They are called *elementary* in the sense that when taken alone, we know how to design stabilizing control laws. The different subsystems are connected through their boundaries. Examples of possible network configurations are given in Figure 1. Thus, a network of hyperbolic systems can be described as a **graph**. For instance, each elementary hyperbolic subsystem can be identified with an edge of a given graph. At the same time, interactions between the PDEs occur at the graph's vertices.

Internship goal: **The general objective** of this internship is to design stabilizing output-feedback controllers for several classes of **networks of linear hyperbolic systems**.

The scientific objective of the internship is to extend existing results developed for simple network configurations (namely chain structure) to more complex configurations. We will start by considering a gradation in the complexity of the network by focusing on **specific network configurations**: chain, divergence, star, simple trees, one cycle. We plan to tackle the problems very gradually, thus obtaining many intermediate results on specific cases that would prove deeply interesting due to the scarcity of literature on this topic.

For each configuration, we want to obtain conditions that characterize the controllability/observability of the system. Then, we aim to design stabilizing output-feedback control laws. Finally, we want to find the minimum number of actuators/sensors (and their respective position in the network) to guarantee the possibility of controlling/observing. Assuming we now have a fixed number of actuators/sensors greater than this value, we want to know all the **admissible configurations** under which it is possible to design output-feedback controllers.

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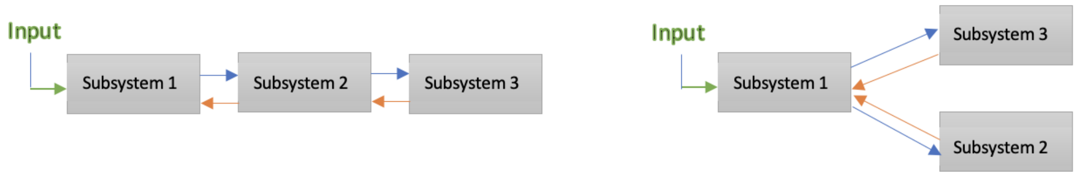

Figure 1: Examples of possible network configurations: chain structure (left), junction/tree (right). The arrows picture possible boundary couplings between the subsystems.

III. Required skills

This internship topic mainly requires good skills in control systems and mathematics (Grandes Ecoles or Master in mathematics/control). Very good results in the engineering curriculum as well as expertise in the topics related to automatic and partial differential equations, will constitute strengths to the proposed subject.

IV. Application

To apply, write an email with your CV and a transcript to J. Auriol and L. Brivadis. The internship should start in February-April 2024. **The funding is guaranteed by the ANR**. This internship can become a thesis (whose funding is already guaranteed by the ANR) starting in October 2024.

V. References

[1] G. Bastin and J.-M. Coron. *Stability and boundary stabilization of 1-d hyperbolic systems.* Springer, 2016.

[2] U. J. F. Aarsnes and R. Shor. Torsional vibrations with bit off bottom: Modeling, characterization and field data validation. *Journal of Petroleum Science and Engineering*, 163:712–721, 2018.

[3] H. Yu and M. Krstic. *Traffic Congestion Control by PDE Backstepping.* Springer, 2023.

[4] J. Auriol and F. Di Meglio. Robust output feedback stabilization for two heterodirectional linear coupled hyperbolic PDEs. *Automatica*, 115:108896, 2020.

[5] R. Curtain and H. Zwart. *An introduction to infinite-dimensional linear systems theory*, volume 21. Springer, 2012.