Model order reduction for hybrid systems
Application to power electronics

Pauline Kergus

1 Context and objectives

A hybrid system is a finite collection of dynamical systems, indexed by a set of discrete modes, i.e. taking their value in a finite set:

$$\begin{cases} \dot{x}=f_q(x,u) \\ y=g_q(x,u) \end{cases} \tag{1}$$

Each dynamic system is governed by a set of differential equations. The change from one mode to another can be externally imposed or depend on the value of the continuous state, and is orchestrated by a switching law. A change of discrete mode leads to a change in the continuous dynamics, and possibly a jump in the state of the system, described by a *reset map*.

In the case of switched affine systems, the dynamic subsystems are linear and the mode changes are controlled externally. This sub-category of hybrid systems is widely studied because, although it is the simplest kind of hybrid systems, it can be used to represent a wide range of applications, particularly in the field of electrical engineering (power converters in energy networks in particular).

Consequently, the analysis and control of these switched linear systems (SLS) are established topics, see [1], [2]. In particular, stability criteria in the form of linear matrix inequalities (LMI) [3][4][5][6] have been developed from Lyapunov theory for the stability analysis of hybrid systems with time-dependent switching laws. In parallel, the extension of Lyapunov stability to the class of discontinuous systems presented in [7] has led to the development of several approaches for the synthesis of stabilising switching laws for switching affine systems. In particular, in [8] a stabilising switching law was proposed based on the existence of a Lyapunov function common to all subsystems. In addition, the switching frequency was limited by using a variable-width hysteresis. Less restrictive stabilisation conditions have recently been proposed in [9][10] using non-quadratic and switched Lyapunov functions. Recently, the problem of synthesising stabilising commutation laws while taking into account the effect of sampling and parametric uncertainties has been increasingly studied [11][12][13][14]. In order to apply these techniques, identification and model reduction of hybrid systems are the subject of an abundant literature, see [15][16][17] for identification and [18][19] for model reduction, as well as the references therein. These techniques differ from traditional methods for model reduction or identification of LTI systems as they are not limited to obtaining continuous submodels, but take into account their complex interactions through the discrete modes.

Identification techniques for hybrid systems focus on obtaining structured input-output representations of a given order from noisy experimental data: piecewise auto-regressive exogenous (PWARX) models [20][21][22] or switched auto-regressive exogenous (SARX) models [23]. A few techniques have been proposed to obtain state-space representations in the multivariable case [24][25][26][27], but they are based on the identification and then fusion of linear subsystems, implying strong assumptions about the subsystems (same order and/or same basis in particular). The model reduction techniques developed for hybrid systems make it possible to obtain state-space representations and are naturally suited for the multivariable case. They also have the advantage of guiding the choice of the order of the reduced system, but they are based on knowledge of the underlying high-order model.

In this context, the main objective of this thesis is to propose an innovative approach combining system identification and model reduction to obtain SLS models from real data. This thesis will be based on a review of existing methods for hybrid systems, both in system identification and model reduction, highlighting their advantages and limitations.

This thesis has an important applicative objective as it should be applied in the field of power electronics, where obtaining reduced models is crucial for stability analysis in converter networks for instance. Indeed, the use of detailed models for all the components of such networks results in great numerical complexity, making it difficult to simulate and analyse long-term stability and to simulate complex systems. Model reduction is therefore a promising avenue [28][29][30][31] for the stability analysis of energy networks dominated by converters. To our knowledge, the reduced models obtained are continuous only and not hybrid for this type of application. The method proposed during this thesis would therefore be innovative in the field of power electronics and would make it possible to obtain finer models reflecting the fast dynamics due to switching. Indeed, fast dynamics have an impact on slow dynamic power loops, and neglecting them can lead to questionable results [32].

2 Expected contributions

This thesis aims to provide methodological tools for modelling hybrid systems, at the crossroads between system identification and model reduction, making it possible to obtain reduced-order models from real data. This is the innovative aspect of the proposed work. To this end, it is planned to work on data-based reduction methods, mainly the Loewner interpolation framework [33], recently extended to SLS [34]. The aim will be to extend this method in order to obtain a reduced model from noisy data and to quantify the approximation error of the resulting model.

1. Initially, it will be necessary to define generalised transfer functions based on [34][35], as done for bilinear systems in [36]. These theoretical concepts are at the heart of the implementation of data-based model reduction methods because it is these generalised transfer functions that are interpolated, making it possible to extend the linear case to non-linear systems on a case-by-case basis.

2. It will then be necessary to estimate the moments of these generalised transfer functions from data, as for bilinear systems in [37], before interpolating them. To do this, it will be necessary to identify the jumps and estimate the discrete state of the system from the data, taking inspiration from the techniques developed in hybrid system identification, see [27] and references therein.

3. Finally, it will be necessary to work on obtaining guarantees on the accuracy of the resulting models. One way of doing this consists in exploiting the similarities between LPV systems (linear systems with varying parameters) and SLSs: while LPV systems are usually subject to slow, continuous variations in their parameters, an SLS undergoes non-smooth, abrupt changes. This correspondence between SLSs and LPV systems is exploited in [38][39][40] to lay the foundations of SLS realisation theory, which will be used and developed in this work to study the consistency of the proposed approach.

These modelling tools will be developed in order to be able to apply advanced control methods to hybrid systems interconnected in large-scale networks, in order to overcome the associated numerical complexity. In particular, this thesis is a continuation of the work carried out in the CODIASE group on the robust control of switched affine systems applied to DC-DC converters [41]. Although this work makes it possible to guarantee stability and performance, in practice it is used on low-dimensional applications. In order to extend these techniques, to more complex converter architectures in particular, the method proposed in this thesis should obtain reduced models that are better suited to the synthesis of robust control laws.

Beyond the expected methodological advances, the applicative dimension of this thesis is equally important. In particular, in the context of the LAPLACE laboratory, the proposed method could be applied to numerous electrical engineering problems (control of networked converters, unit commitment, etc.). While power conversion is one of the major applications considered by the automatic control community for hybrid control and analysis techniques, these techniques remain rarely applied by power electronics specialists, who mainly use well-established techniques based on the use of average and non-hybrid models [42]. On this subject, however, we should mention [43] on stability analysis based on the use of a hybrid model. Furthermore, to our knowledge, implementations and experimental validations of hybrid control laws on converters remain fairly rare [44][45][46][47][48] despite promising results. By proposing to obtain hybrid models that are reduced in size and therefore easier to manipulate, it is possible to hope to facilitate the use of hybrid techniques in the field of power electronics. In the long term, this work is part of a wider project to transfer hybrid system control and analysis methods from the automatic control community to the electrical engineering community, particularly in the field of power electronics.

3 PhD organisation

The thesis will be carried out at LAPLACE, in the CODIASE group. The thesis will be directed by Pauline Kergus (CR CNRS, LAPLACE, groupe CODIASE), and co-supervised by Zohra Kader (MdC INPT, LAPLACE, groupe CODIASE). Pauline Kergus has extensive experience with the Loewner framework, its non-linear extensions and its use on noisy data in the linear case, and Zohra Kader will contribute with her expertise on switched systems and their control.

The following organisation is proposed for the 36 months of the thesis:

- M0 to M6: Bibliography on model identification and reduction for hybrid systems
- M6 to M18: Methodological development
  - Definition of generalized transfer functions for hybrid systems
  - Data-based estimation of the moments of these transfer functions
  - Accuracy guarantees
- M18 to M30: Application to power electronics
  - Benchmarks considered: IEEE Power Electronics test feeders [49], examples developed in [29][30][31].
  - Obtaining reduced models and validation in simulation
  - Use of reduced models for hybrid control and/or stability analysis
- M30 to M36: thesis writing and preparation of the defence.

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