Polynomial optimization for tokamak control

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General presentation : Fossil fuels (oil, gas, coal) currently make up 85 % of global primary energy sources, despite their upcoming shortage within a few decades and the greenhouse effect of the CO2 resulting from their combustion. **Thermonuclear fusion** controlled by magnetic confinement, as done in tokamaks, is one of the existing possibilities to sustainably answer our needs in energy. Control problems related to tokamaks are becoming more and more decisive for the success of magnetic fusion, and will be crucial for ITER. Feedback control of the main macroscopic parameters of the plasma is currently well understood in the existing tokamaks. However, **controlling internal radial profiles of the plasma** remains a challenge, and is necessary to ensure both secure tokamak operation and performances of the plasma regimes. Operating the tokamak in "advanced" H mode, as currently considered for ITER, needs a regulation of transport internal barriers (local magneto-hydrodynamic phenomena) to enable a significant increase of the plasma core energy and thus boost combustion. This aim of **optimizing combustion** requires to explicitly account for flow terms, often described under the form of polynomials.

Polynomial optimization (a.k.a. SoSP : Sum of Squares Programming) recently found new application to the study of Partial Differential Equations (PDEs) through two approaches : ***(1)*** the **moment-SoS hierarchy** resorts to occupation measures to generically represent solutions to a PDE, at the price of assumptions not always in line with physical reality. However, promising preliminary results have been published on the study of transport PDEs. ***(2)*** Certification of **functional inequalities** (Lyapunov, Wirtinger...) has proved decisive for stability analysis, with a strong potential application to tokamak control. A downside of this approach is that the involved Lyapunov functionals are a data of the problem, and SoSP is only used to validate their properties, while the moment-SoS hierarchy automates the search for Lyapunov functions [1].

The thesis will have **combustion control in a tokamak plasma** as its main theme, in an "advanced" operation scenario, and will have to use and improve existing results on control of PDEs, and more particularly nonlinear (reaction-)diffusion equations. In particular, generalizing existing results to various tokamak control problems, such as algorithmic control synthesis, without resorting to heuristic Lyapunov functionals, will be a starting point of the study. This will then be complemented with harder questions more difficult questions on inverse problems (e.g. computing regions of attraction) or control of other **parabolic PDEs**, which will lead to focus on the more generic mathematical framework of occupation measures.

International mobility : 3 months secondment at Lausanne's Institute of Technology (EPFL – Switzerland) within the Swiss Plasma Center (A. Fasolini, F. Felici, O. Sauter), with possible interaction with the Automatic Control Laboratory (C.N. Jones, G. Ferrari-Trecate) and the Risk Analytic and Optimization Chair (D. Kuhn).

Societal and environmental impact : Controlled thermonuclear fusion has the potential to meet the global need for **sustainable energy**. The energy generated by the fusion of deuterium and tritium (isotopes of hydrogen that are extracted from water and earth crust) can be performed **harmlessly** (no direct radioactive waste is produced and the structure radioactivity rapidly decays). Fusion infrastructures that use plasma magnetic confinement, as tokamaks, can thus be considered as a **major carbon-free energy source** for future. Nevertheless, far reaching challenges still need to be leveraged before tokamaks can be reliably exploited.

The ITER [2] tokamak, an **international project** implying seven members (EU, Russia, USA, Japan, China, Korea and India), should start its operation during the next decade. It is planned to produce 500 MW from 50MW input power. The feedback control of the main macroscopic parameters of the plasma, such as its position and shape, total current and density, is currently quite well understood in existing tokamaks. However, the control of the plasma dynamics as well as radial profiles (1-D distributions) is still in its infancy. This control is crucial for robust stability and **high performance** operation of the tokamak.

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1. At least in the case of ordinary differential equations, PDEs currently posing some additional challenges.
2. https://www.iter.org/

Methodology : The proposed approach is based on SoSP and the moment-SoS hierarchy, and can be considered as an instance of the generic category of **lift-and-project** methods. First, these techniques consist in recasting a (difficult) given problem, under the form of a more structured one, at the price of increasing the ambient space dimension. A classical example is support vector machines (SVM) : a SVM is a linear object (a hyperplane) used to separate data sample into two groups ; however, some data samples are not linearly separable (see the plot on the left of Figure 1). The problem is then **recast** in a larger space, in which data become linearly separable (plot on the right of Figure 1).

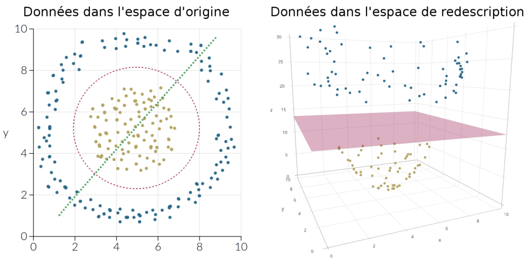

FIGURE 1 – Lifting for SVM

Sometimes, the dimension of the recasting space can be particularly large (or even infinite), which would render any computation intractable. Thus, in a second step, left-and-project techniques include a **projection**, which consists in approximating the recast problem with a tractable relaxation in a smaller space. The moment-SoS hierarchy follows the exact same principle, with infinite dimensional recasting spaces : a space of (Lyapunov) functions or (occupation) Borel measures. The projection step consists in looking for functions under the form of sums of squares of polynomials with bounded degree. In the case of the tokamak, already represented by an infinite dimensional equation (PDE), the lifting step mostly consists in looking for another infinite dimensional formulation with more structure, which is made possible by the frameworks of occupation measures and measure valued solutions.

How to apply : Follow the link `https://adum.fr/as/ed/voirproposition.pl?langue=&matricule_prop=49243&site=edeeats` and contact the PhD supervisors [3,4].

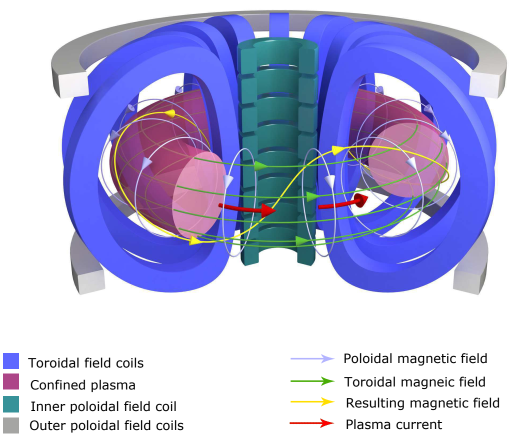

FIGURE 2 – Schematic of the workings of a tokamak.

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